F-theory and Particle Physics

Eran Palti

(Ecole Polytechnique, Paris)

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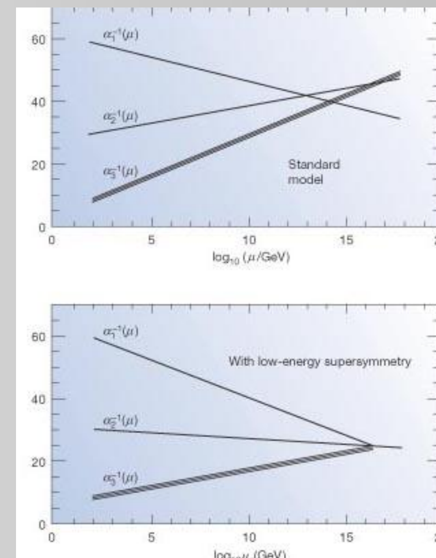
Unification of fields in the UV has been a recurring theme in particle physics

Forces

Electric + Magnetic + Weak ✓

SU(3)xSU(2)xU(1) → SU(5)



Particles

Atoms → Hadrons+leptons → Quarks+leptons → UV

$(3,2)_{1/6}+(\bar{3},1)_{-2/3}+(1,1)_{1}$ = 10 of SU(5)

$(1,2)_{-1/2}+(3,1)_{+1/3}$ = $\bar{5}$ of SU(5)

The imminent discovery of the Higgs at 125 GeV has substantial implications if we insist on keeping unification and naturalness

Supersymmetry remains the leading candidate consistent with these principles

The unification aspects of supersymmetry are striking:

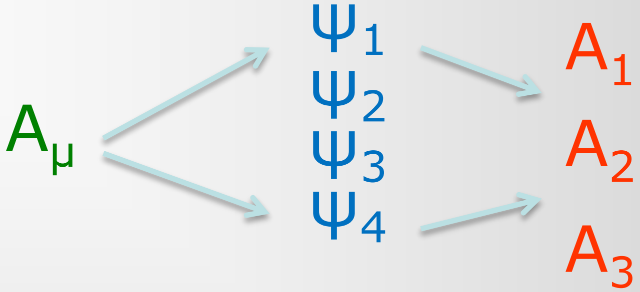
Scalars ⟷ Fermions ⟷ Gauge vectors

Supersymmetry at the TeV scale is minimal (N=1)

Expect enhanced supersymmetry in the UV (N=2, N=4)

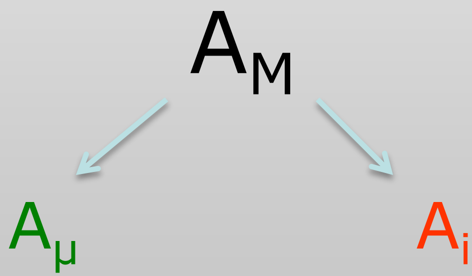
Strings see maximal supersymmetry in the UV

Maximal 4-dimensional SUSY is N=4 Super Yang-Mills



Everything is unified into gauge fields: SM fields are gauginos!

Supersymmetry is closely related to extra dimensions



Maximal N=4 4-dimensional SUSY implies 10 dimensions

Half N=2 4-dimensional SUSY implies 6 dimensions

So what are the gauge groups that give the SM fields as gauginos?

First unify into GUT multiplets

$(3,2)_{1/6}+(\bar{3},1)_{-2/3}+(1,1)_{1}$ =10 of SU(5)

$(1,2)_{-1/2}+(3,1)_{+1/3}$ $=\bar{5}$ of SU(5)

Then since gauginos in adjoint look for groups that can embed also with conjugate N=2 partners

SU(6) → SU(5)xU(1)

35 → $24+(5+\bar{5})+1$

SO(10) → SU(5)xU(1)

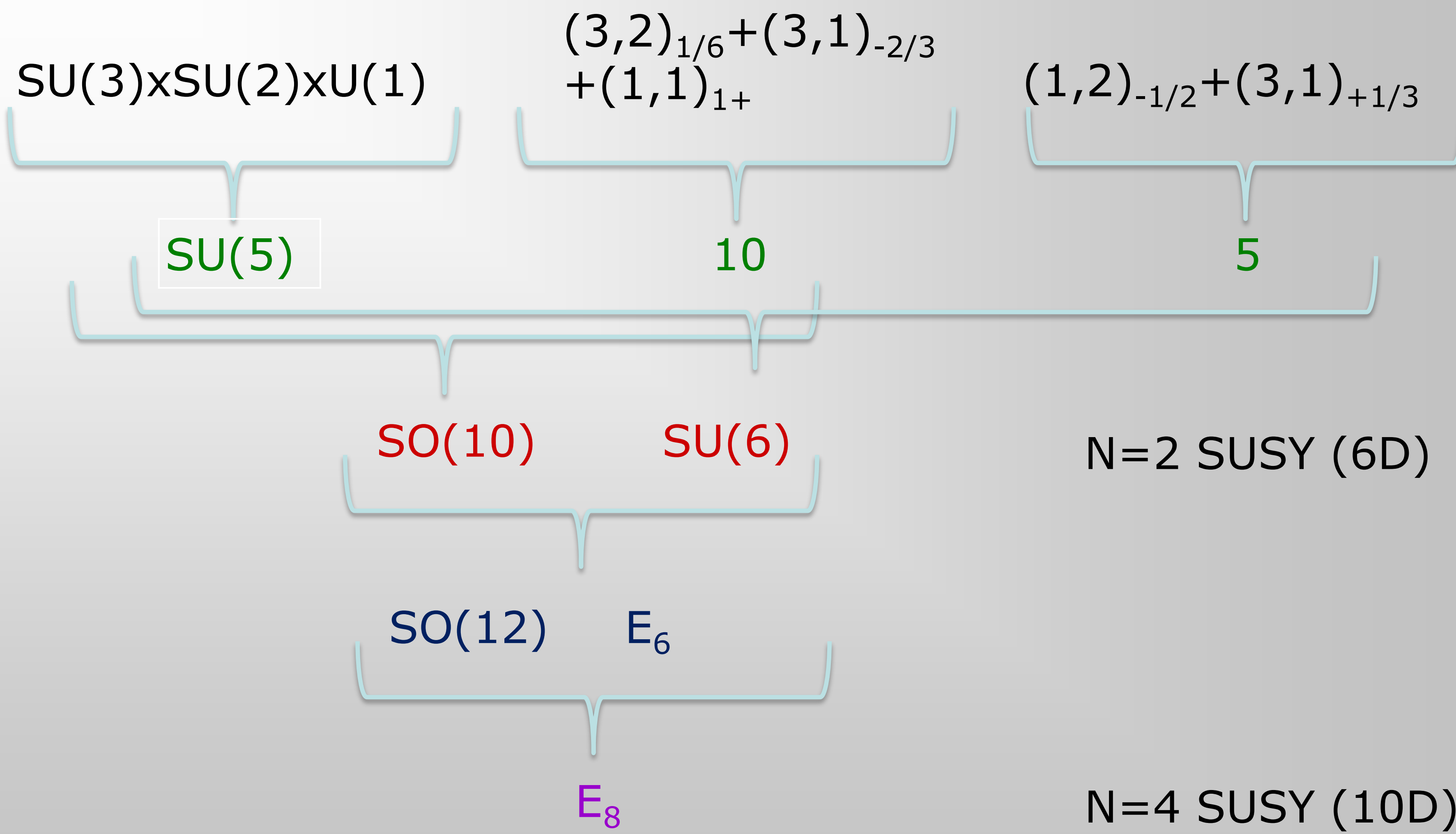
45 → $24+(10+\overline{10})+1$

What about interactions such as Yukawa couplings?

Should arise as cubic couplings of the UV gauge field

$$
\begin{aligned}
SO(12) \quad &\supset \quad SU(5) \times U(1)_a \times U(1)_b \,, && (2.1)\\
\mathbf{66} \quad &\to \quad \mathbf{24}^{(0,0)} \oplus \mathbf{1}^{(0,0)} \oplus \mathbf{1}^{(0,0)} \oplus (\mathbf{5} \oplus \bar{\mathbf{5}})^{(-1,0)} \oplus (\mathbf{5} \oplus \bar{\mathbf{5}})^{(1,1)} \oplus (\mathbf{10} \oplus \bar{\mathbf{10}})^{(0,1)} \,, \\
E_6 \quad &\supset \quad SU(5) \times U(1)_{a'} \times U(1)_{b'} \,, && (2.2)\\
\mathbf{78} \quad &\to \quad \mathbf{24}^{(0,0)} \oplus \mathbf{1}^{(0,0)} \oplus \mathbf{1}^{(0,0)} \oplus \mathbf{1}^{(-5,-3)} \oplus \mathbf{1}^{(5,3)} \\
&\oplus (\mathbf{5} \oplus \bar{\mathbf{5}})^{(-3,3)} \oplus (\mathbf{10} \oplus \bar{\mathbf{10}})^{(-1,-3)} \oplus (\mathbf{10} \oplus \bar{\mathbf{10}})^{(4,0)} \,.
\end{aligned}
$$

So what is super grand unification?



E8 structure underlying the gauge groups+matter+interactions of the SM

In string theory N=4/10D/E8 structure comes out of

Heterotic String | F-theory (IIB) | M-theory (IIA)

Differ by the dimension on which gauge fields localise

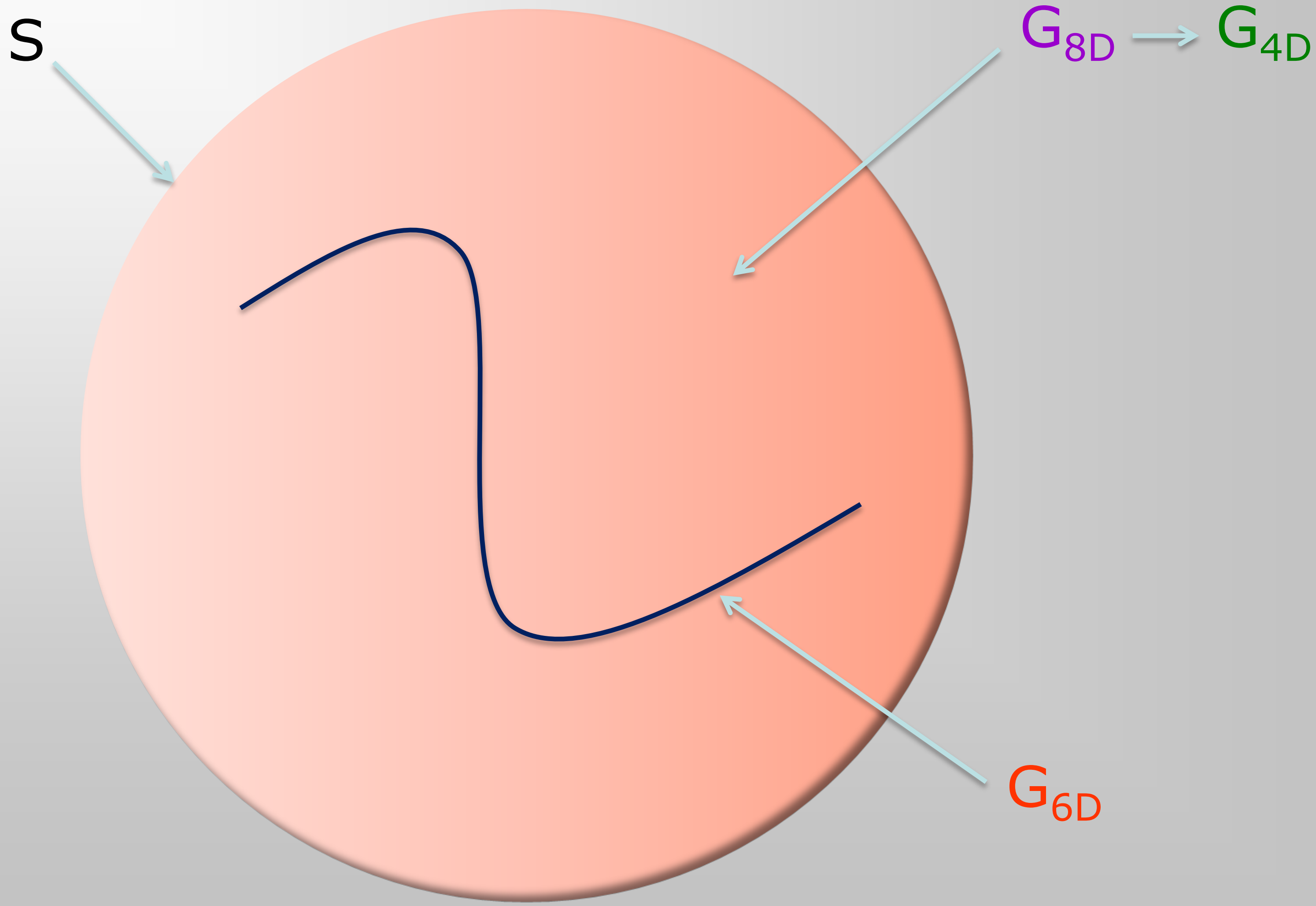
F-theory is strongly coupled IIB and has 7-branes with gauge fields

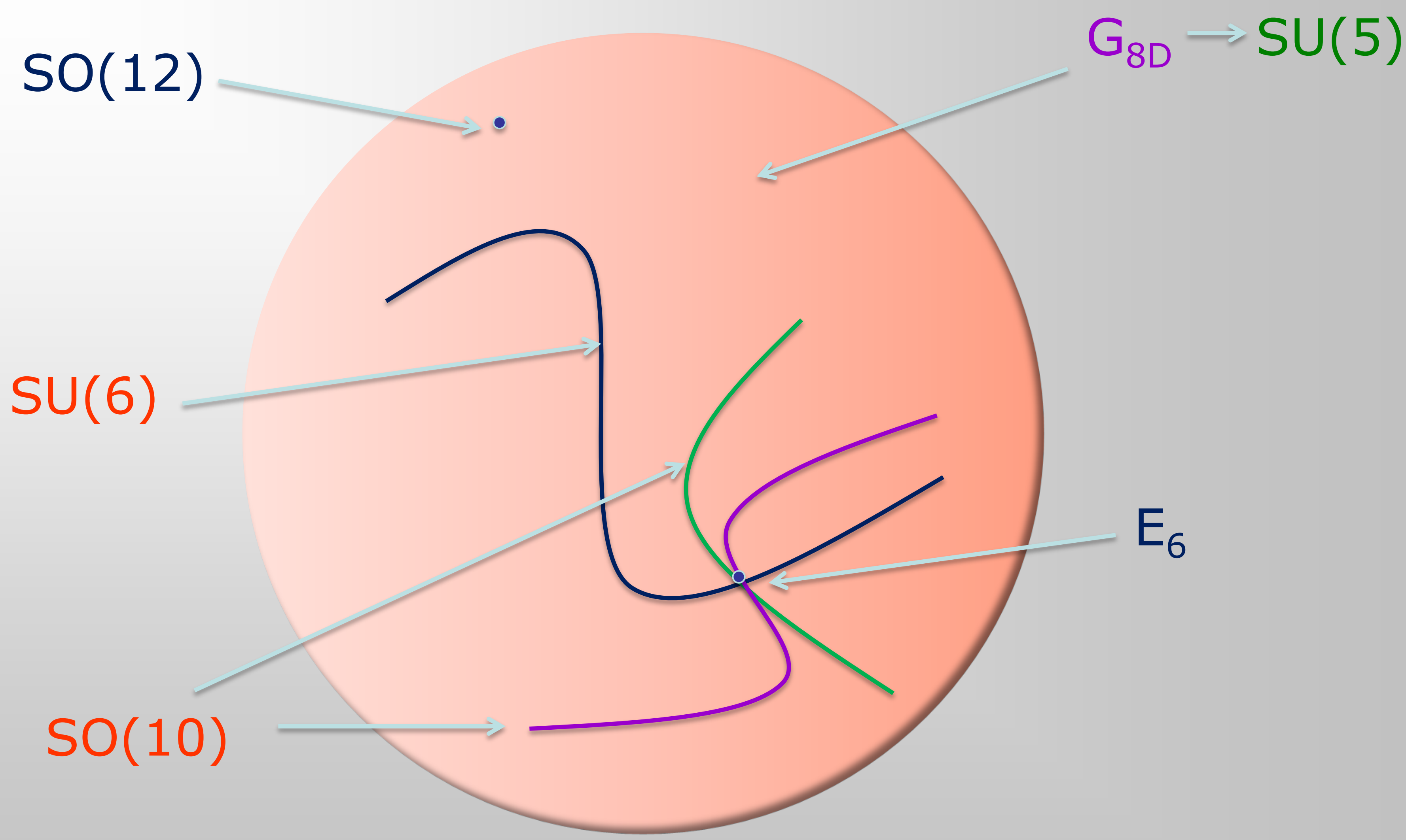
10D SYM → 8D SYM $A_m \rightarrow A_m$ $A_{(9,10)} \rightarrow \Phi$

Can write the field content in terms of 4-dimensional N=1 multiplets

$$
\begin{aligned}
\mathbf{A}_{\bar{m}} &= \{A_{\bar{m}}, \psi_{\bar{m}}, \mathcal{G}_{\bar{m}}\} \ , \\
\mathbf{\Phi}_{mn} &= \{(\varphi_H)_{mn}, \chi_{mn}, \mathcal{H}_{mn}\} \ , \\
\mathbf{V} &= \{\eta, A_\mu, \mathcal{D}\} \ ,
\end{aligned}
$$

The Higgs field breaks the 8-dimensional gauge group but vanishes on subspaces where the gauge group enhances

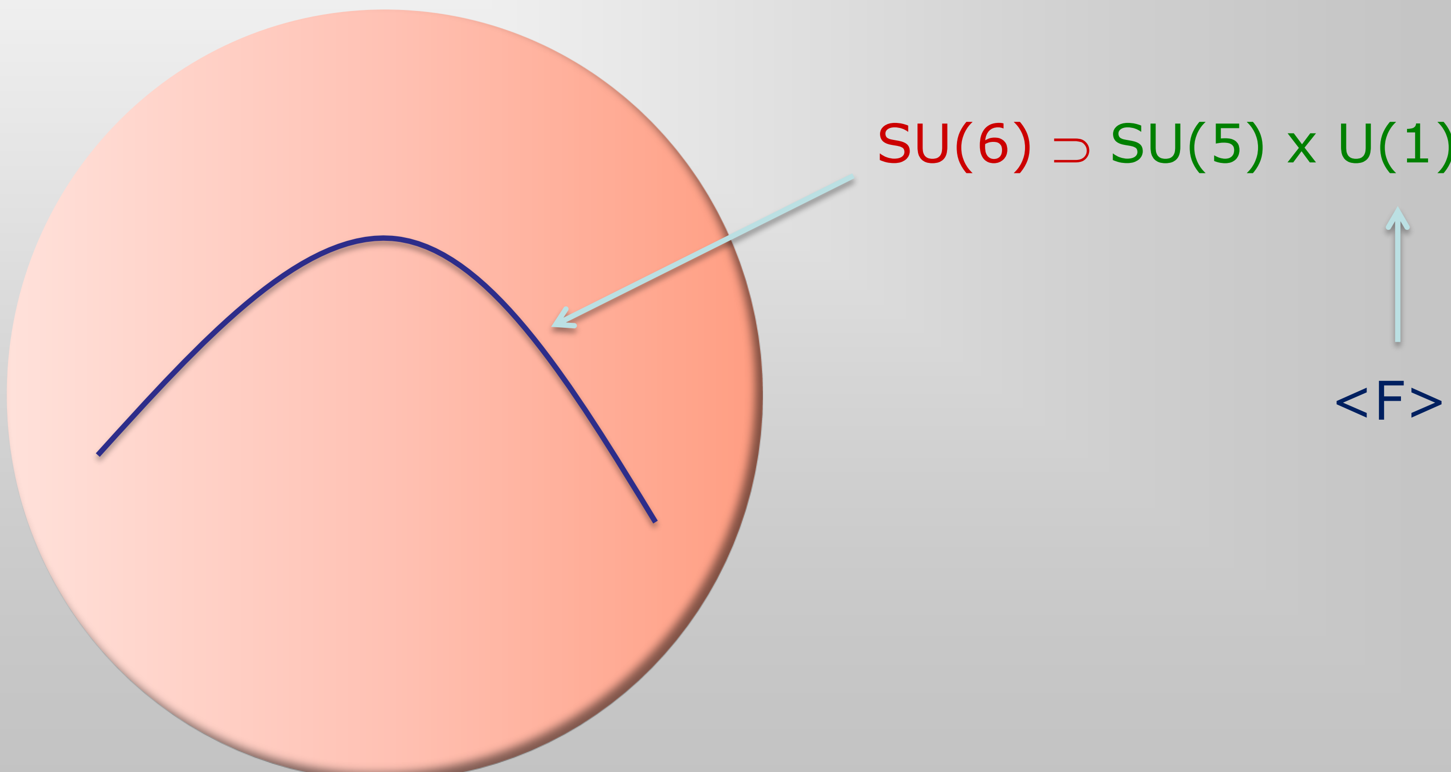




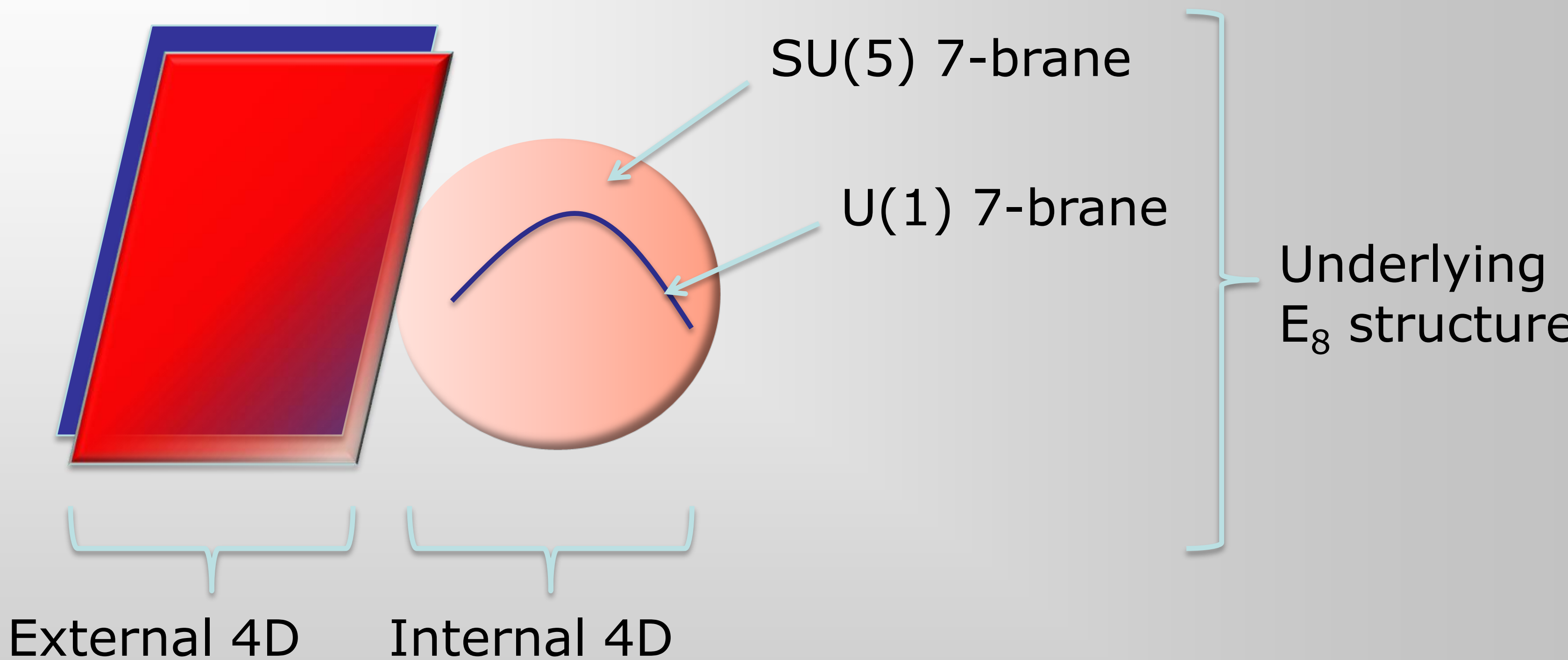
Beasly, Heckman, Vafa `08;

So far fields are N=2 so not chiral, to generate chirality must turn on background flux along the matter curves

Solutions to the Dirac equation in the presence of a background gauge field are chiral



Can think of this picture in terms of intersecting 7-branes



F-theory describes this elegantly using an elliptically fibered CY 4-folds with A-D-E singularities

F-theory global model building amounts to studying singular elliptically fibered CY 4-folds in the presence of 4-form flux

Blumenhagen, Grimm, Jurke, Weigand, Krause, Hebecker, Mayrhofer, Kerstan, Schafer-Nameki, Marsano, Saulina, Collinucci, Savelli, Braun, Valandro …

The underlying E_8 structure described by the Tate form of the elliptic fibration

$$x^3 - y^2 - xy\alpha_1 + x^2\alpha_2 - y\alpha_3 + x\alpha_4 + \alpha_6 = 0 \; .$$

Can study explicitly the singularity structure over subspaces

$$\alpha_1 = b_5 \; , \;\; \alpha_2 = b_4 z \; , \;\; \alpha_3 = b_3 z^2 \; , \;\; \alpha_4 = b_2 z^3 \; , \;\; \alpha_6 = b_0 z^5 \; ,$$

$$\Delta = -z^5 \left[P_{10}^4 P_5 + z P_{10}^2 \left(8 b_4 P_5 + b_5 R \right) + \mathcal{O}\left(z^2\right) \right] \; ,$$

z=0 ⟶ SU(5)

What does this picture imply for low-energy particle physics and string phenomenology?

Small selection from an active area…

Dudas, EP `10;

Camara, Dudas, EP `11;

Constrained structure of additional U(1) symmetries

$SU(5) \times U(1)^4$

E_8

Additional U(1) symmetries are often used in BSM physics, but the embedding of matter+symmetries into E_8 is very restrictive

At a generic point in S the gauge group is broken to

$$E_8 = SU(5)_{GUT} \times SU(5)_{\perp} \to SU(5)_{GUT} \times U(1)^4$$

$$\mathbf{248} \;\to\; (\mathbf{24},\mathbf{1}) \oplus (\mathbf{1},\mathbf{24}) \oplus (\mathbf{10},\mathbf{5}) \oplus (\bar{\mathbf{5}},\mathbf{10}) \oplus (\overline{\mathbf{10}},\bar{\mathbf{5}}) \oplus (\mathbf{5},\overline{\mathbf{10}})$$

The matter representations decompose according to U(1) charges

$$t_1 = (1,0,0,0,0)\ ,\ t_2 = (0,1,0,0,0)\ ,\ \ldots \qquad t_1 + t_2 + t_3 + t_4 + t_5 = 0$$

$$\Sigma_{\mathbf{10}\oplus\overline{\mathbf{10}}} \;:\; t_i \qquad \Sigma_{\mathbf{5}\oplus\bar{\mathbf{5}}} \;:\; -t_i - t_j \qquad \Sigma_{\mathbf{1}} \;:\; \pm(t_i - t_j)$$

Study selection rules from additional symmetries

Beasley, Heckman, Vafa, Donagi, Wijnholt, Ibanez, Font, Schafer-Nameki, Marsano, Saulina, Dolan, Weigand, Grimm, Kerstan, Conlon, Dudas, EP, Aparicio, Marchesano, Leontaris, King, Ross, Callaghan, Ludeling, Nilles, Stephan, …

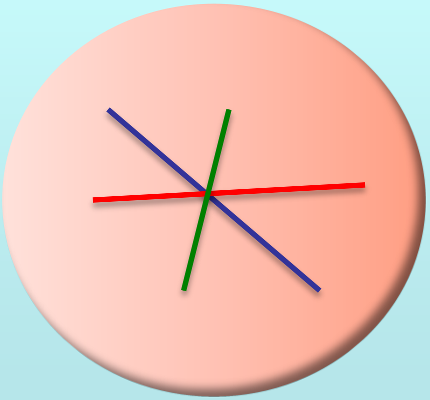
Example application at the GUT level:

Dudas, EP `10;

Field	Curve	Charges/Orbit
Chiral spectrum		
$\mathbf{5}_{H_u}$	$\mathbf{5}_{H_u}$	$-t_1 - t_2$
$\bar{\mathbf{5}}_{H_d}$	$\bar{\mathbf{5}}_5$	$t_3 + t_5$
$\mathbf{10}_t$	$\mathbf{10}_1$	$\{t_1, t_2\}$
$\mathbf{10}_c$	$\mathbf{10}_3$	t_4
$\mathbf{10}_u$	$\mathbf{10}_2$	t_3
$\bar{\mathbf{5}}_b$	$\bar{\mathbf{5}}_2$	$\{t_1 + t_4, t_2 + t_4\}$
$\bar{\mathbf{5}}_s$	$\bar{\mathbf{5}}_1$	$\{t_1 + t_3, t_2 + t_3\}$
$\bar{\mathbf{5}}_d$	$\bar{\mathbf{5}}_4$	$t_3 + t_4$
N_1	$\bar{\mathbf{1}}_6$	$-t_4 + t_5$
N_2	$\bar{\mathbf{1}}_5$	$-t_3 + t_5$
N_3	$\bar{\mathbf{1}}_3$	$\{-t_1 + t_5, -t_2 + t_5\}$
X_1	$\bar{\mathbf{1}}_4$	$-t_3 + t_4$
X_2	$\mathbf{1}_2$	$\{t_1 - t_4, t_2 - t_4\}$
X_3	$\bar{\mathbf{1}}_1$	$\{-t_1 + t_3, -t_2 + t_3\}$
Non-Chiral spectrum		
-	$\mathbf{10}_4$	t_5
-	$\mathbf{5}_3$	$\{-t_1 - t_5, -t_2 - t_5\}$
-	$\mathbf{5}_6$	$-t_4 - t_5$
-	$\mathbf{1}_7$	$t_1 - t_2$

Chiral interactions	
$\mathbf{5}_{H_u}\mathbf{10}_i\mathbf{10}_j$	$\begin{pmatrix} \epsilon_2^2\epsilon_{\bar{4}}^2 & \epsilon_2^2\epsilon_{\bar{4}} & \epsilon_2\epsilon_{\bar{4}} \\ \epsilon_2^2\epsilon_{\bar{4}} & \epsilon_2^2 & \epsilon_2 \\ \epsilon_2\epsilon_{\bar{4}} & \epsilon_2 & 1 \end{pmatrix}$
$\bar{\mathbf{5}}_{H_d}\bar{\mathbf{5}}_i\mathbf{10}_j$	$\begin{pmatrix} \epsilon_2^2\epsilon_{\bar{4}}^2 & \epsilon_2\epsilon_{\bar{4}}^2 & \epsilon_2\epsilon_{\bar{4}} \\ \epsilon_2^2\epsilon_{\bar{4}} & \epsilon_2\epsilon_{\bar{4}} & \epsilon_2 \\ \epsilon_2\epsilon_{\bar{4}} & \epsilon_{\bar{4}} & 1 \end{pmatrix}$
$K \supset \mathbf{5}_{H_d}\bar{\mathbf{5}}_i N_j$	$\begin{pmatrix} \epsilon_2 & 1 & \epsilon_{\bar{1}}\epsilon_2 \\ \epsilon_{\bar{1}}\epsilon_2 & \epsilon_{\bar{1}} & \epsilon_{\bar{1}}\epsilon_{\bar{1}}\epsilon_2 \\ 1 & \epsilon_{\bar{1}}\epsilon_{\bar{4}} & \epsilon_{\bar{1}} \end{pmatrix}$
$\beta\mathbf{5}_{H_u}\bar{\mathbf{5}}_i$	$(\epsilon_4\epsilon_2^2, \epsilon_4\epsilon_2, \epsilon_2)$
$\mathbf{5}_{H_u}\bar{\mathbf{5}}_i N_j$	0
MN_iN_j	0
$\bar{\mathbf{5}}_i\bar{\mathbf{5}}_j\mathbf{10}_k$	0
$\mathbf{10}_i\mathbf{10}_j\mathbf{10}_k\bar{\mathbf{5}}_l$	0
$\mu\mathbf{5}_{H_u}\bar{\mathbf{5}}_{H_d}$	0

Enhanced calculability and generality from matter localisation



New way to understand doublet-triplet splitting

The 4-dimensional fields decent from the 8-dimensional ones as

$$\begin{aligned} \mathbf{A}_{\bar{m}} &= \{A_{\bar{m}}, \psi_{\bar{m}}, \mathcal{G}_{\bar{m}}\} \ , \\ \mathbf{\Phi}_{mn} &= \{(\varphi_H)_{mn}, \chi_{mn}, \mathcal{H}_{mn}\} \ , \\ \mathbf{V} &= \{\eta, A_\mu, \mathcal{D}\} \ , \end{aligned}$$

Interactions between fields come from the coupling

$$\int_{M_8} \mathrm{Tr}\left[F \wedge \Phi\right] \supset \int_{M_8} \mathrm{Tr}\left[A \wedge A \wedge \Phi\right]$$

Coefficient of a cubic superpotential operator can be calculated as a wavefunction overlap integral

$$\Psi_{8D} = \phi_{\mathrm{4D}} \times \psi_{\mathrm{int}}$$

$$\int_{4D \times S} \Psi^1 \Psi^2 \Psi^3 = \int_{4D} \phi^1 \phi^2 \phi^3 \left(\int_S \psi^1 \psi^2 \psi^3 \right)$$

In a Higgs and Flux background

$$\langle \varphi_H \rangle = M_K R\, m_i^a z_i Q_a dz_1 \wedge dz_2 + \dots \ ,$$

$$\langle A \rangle = -M_K \,\mathrm{Im}(M_{ij}^a z_i d\bar{z}_j) Q_a + \dots \ ,$$

The wavefunctions of massless modes satisfy the equation

$$\mathbb{D}^- \Psi = 0 \ ,$$

$$\mathbb{D}^\pm = \begin{pmatrix} 0 & D_1^\pm & D_2^\pm & D_3^\pm \\ -D_1^\pm & 0 & -D_3^\mp & D_2^\mp \\ -D_2^\pm & D_3^\mp & 0 & -D_1^\mp \\ -D_3^\pm & -D_2^\mp & D_1^\mp & 0 \end{pmatrix} , \qquad \Psi = \begin{pmatrix} \eta \\ \psi_{\bar{1}} \\ \psi_{\bar{2}} \\ \chi \end{pmatrix} ,$$

$$D_i^- \equiv \partial_i - \frac{1}{2} q_a \bar{M}_{ji}^a \bar{z}_j \qquad D_i^+ \equiv \bar{\partial}_i + \frac{1}{2} q_a M_{ji}^a z_j$$

$$D_3^- \equiv -R\, q_a \bar{m}_i^a \bar{z}_i \qquad D_3^+ \equiv R\, q_a m_i^a z_i \ .$$

And have solutions of the form

$$\varphi = f\left(-\hat{\xi}_{1,2} z_1 + \hat{\xi}_{1,1} z_2\right) e^{-p_1|z_1|^2 - p_2|z_2|^2 + p_3 \bar{z}_1 z_2 + p_4 \bar{z}_2 z_1} \ ,$$

Can use wavefunctions profile to study Yukawa couplings for example

Heckman, Vafa, Cecotti, Ibanez, Font, Conlon, Camara, Dudas, EP, Hayashi, Kawano, Tsuchiya, Watari, Aparicio, Marchesano, Martucci, Leontaris, King, Ross, Callaghan, …

Will present discussion regarding doublet-triplet splitting

Superpotential of a SUSY SU(5) GUT theory

$$W \supset Y_{ij}^u H^u Q_i U_j + Y_{ij}^d H^d \left(Q_i D_j + L_i E_j\right) + \hat{Y}_{ij}^u T^u \left(Q_i Q_j + U_i E_j\right) + \hat{Y}_{ij}^d T^d \left(Q_i L_j + U_i D_j\right) + M T^u T^d \,.$$

Crucial to understand where the mass term comes from?

Also the nature of the triplet couplings for proton decay

$$W \supset \frac{\hat{Y}_{ij}^u \hat{Y}_{kl}^d}{M} \left(Q_i Q_j Q_k L_l + U_i E_j U_k D_l\right) \,.$$

In F-theory doublet-triplet splitting is induced by turning on background flux in the hypercharge direction

The chiral spectrum is given by the integrated flux integers

5

$$
\begin{aligned}
n_{(\mathbf{3},\mathbf{1})_{-1/3}} - n_{(\bar{\mathbf{3}},\mathbf{1})_{1/3}} &= M_5 \ , \\
n_{(\mathbf{1},\mathbf{2})_{1/2}} - n_{(\mathbf{1},\mathbf{2})_{-1/2}} &= M_5 + N \ ,
\end{aligned}
$$

10

$$
\begin{aligned}
n_{(\mathbf{3},\mathbf{2})_{1/6}} - n_{(\bar{\mathbf{3}},\mathbf{2})_{-1/6}} &= M_{10} \ , \\
n_{(\bar{\mathbf{3}},\mathbf{1})_{-2/3}} - n_{(\mathbf{3},\mathbf{1})_{2/3}} &= M_{10} - N \ , \\
n_{(\mathbf{1},\mathbf{1})_{1}} - n_{(\mathbf{1},\mathbf{1})_{-1}} &= M_{10} + N \ ,
\end{aligned}
$$

Where M is flux along U(1) in SU(5)xU(1) and N is along the Hypercharge direction

New solution to the doublet-triplet splitting problem

The nature of triplets is very different to minimal SU(5) GUTs

What is the nature of the triplet `Yukawa` couplings and what are the effects on proton decay?

Camara, Dudas, EP `11;

Y_{ij} Yukawa couplings calculated by wavefunction overlaps of 3 massless modes

$\hat{Y}_{ij}$ Triplet coupling calculated by overlaps of 2 massless modes with one massive mode

Massive modes are created by appropriate raising operators in the 8-dimensional theory

$$\Psi_{P,(n,m,l)} = \frac{\left(i\tilde{D}_1^+\right)^n \left(i\tilde{D}_2^-\right)^m \left(i\tilde{D}_3^-\right)^l}{\sqrt{m!n!l!}\left(-\lambda_1\right)^{n/2} \lambda_2^{m/2} \lambda_3^{l/2}} \Psi_P \ ,$$

$$\left[\tilde{D}_p^+ , \tilde{D}_q^-\right] = -\delta_{pq}\lambda_p \ .$$

Can study the triplet couplings, find comparable to Yukawas

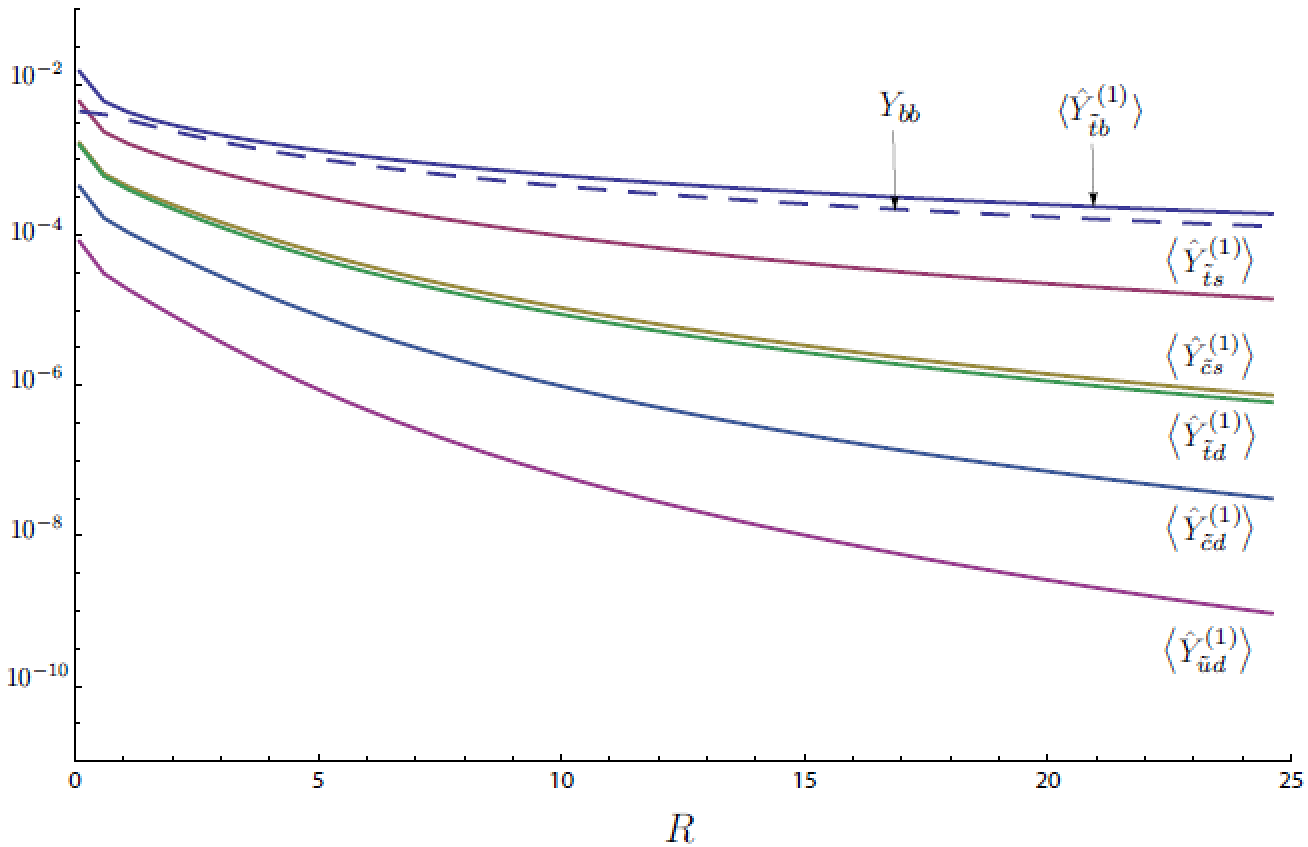


Figure 5: Couplings to the lightest massive vector-like pair of triplets in the first tower of fields localised in the Higgs curve, for flux values $v_1 = 5/6$, $v_2 = 5/4$, $M_1 = 1.6$, $M_2 = 2$, $\gamma = 1.2$, $\varepsilon = 1/10$, $k_{\mathrm{KK}} = 1$ and trivial Landau-level quantum numbers. For reference, we also

Summary

Contemporary string theory model building views the matter and interactions of the MSSM through gauge symmetries localised in varying dimensions

Approach naturally advocates a unification of the gauge+matter+interactions within an E_8 framework

Naturally realised in F-theory models

Implies a restricted but rich sector of BSM physics which can be used to understand long standing puzzles such as proton decay, the quark masses, neutrino masses…

The localisation of interactions implies enhanced calculability allowing for quantitative studies of operator coefficients