A Realistic Unified Gauge Coupling from the Micro-landscape of Orbifold GUTs

A. Hebecker (Heidelberg)

in collab. with Christian Gross (Hamburg) – 0812.4267 (ph)

Outline

- Two-Loop Casimir-Stabilization of $S^1/(Z_2 \times Z_2')$

  [This a straightforward extension of previous work with G.v. Gersdorff to a realistic SUSY model.]

- "Predicting" the 4d gauge coupling at M_{GUT}
- "Uplifting" the stabilized AdS_4 vacuum

Motivation

a) Field-Theoretic

- 5d SU_5 orbifold GUTs are the simplest realistic GUT models
- The 5d-Radius determines the 4d gauge coupling

b) String-Theoretic

- Heterotic orbifold models are among the most realistic string models of the "real world" (→ talks of M. Ratz, H. Nilles, J. Schmidt, ...)
- 5d orbifold GUTs can arise naturally as eff. theories between string- and GUT-scale
- Such "anisotropic orbifolds" may be helpful to overcome the string-scale / GUT-scale problem (Ibanez / Lüst '92 ; Witten '96
A.H. / Trapletti '04 ; Dundee / Raby / Wingerter '08)

The Problem

(We will take a purely 5d-field-theoretic point of view throughout)

Compactification $\Rightarrow$ $$g_4^2 = \frac{g^2}{2\pi R}$$

Proper expansion parameters: $$\frac{g_4^2 N}{16\pi^2} \qquad\qquad \frac{g^2 N}{24\pi^3} \equiv \frac{1}{M}$$

(fundamental scale)

The compactification-relation takes the form:

$$\frac{g_4^2 N}{16\pi^2} = \frac{3}{4} \cdot \frac{1}{MR}$$

$\alpha_{GUT} \sim \frac{1}{25}$ implies $\boxed{MR \simeq 45}$ ("mild hierarchy")

2-loop Casimir Stabilization

(cf. Gersdorff / A.H. '05)

S^1: $$V(R) \sim \frac{1}{R^4} + \frac{g^2}{R^5} \sim \frac{1}{R^4} + \frac{1}{MR^5}$$

- The signs and values of coefficients follow from field content
- Ratio of coefficients determines numerical prefactor in
$$R_{min} \sim \frac{1}{M}$$
- For appropriate field content, the prefactor can be (accidentally) large:
$$R_{min} \gg \frac{1}{M}$$
- If so, higher loops are irrelevant and the minimum is "perturbatively controlled".

2-loop Casimir stabilization

$$S^1/(Z_2 \times Z_2'): \quad V(R) \sim \frac{1}{R^4} + \frac{g^2}{R^5} \ln(\Lambda \cdot R)$$

cutoff

Origin of log-enhancement

- At 1-loop, brane-localized kinetic terms (e.g. F^2) arise
- They are UV-divergent and modify KK-spectrum by terms $\sim g^2$

  (→ Barbieri / Hall / Nomura & many others)

- Calculating the 1-loop Casimir energy with these corrections gives the above "dominant 2-loop effect" (indep. of the UV completion)

- We choose $\Lambda = \Lambda_{max} = M$ $\Rightarrow$ $V(R) \sim \dfrac{1}{R^4} + \dfrac{\ln(MR)}{MR^5}$

SUSY - Breaking

- Of course, $V(R) \equiv 0$ if SUSY is unbroken.

- We assume Scherk-Schwarz SUSY-breaking with an $SU(2)_R$-twist $\omega \ll 1$. $\left(m_{1/2} \sim \omega / R \right)$

- Equivalently, we can think of a no-scale model with radion superfield $T = R + \cdots$. In the presence of $W = W_0$, we find $F_T \neq 0 \qquad \left(F_T \sim \omega \sim \frac{|W_0|}{M_{P,5}^3} \right)$

(cf. Marti / Pomarol; Kaplan / Weiner; Chacko / Luty; Gersdorff / Quiros / Riotto; ...)

- $\boxed{\text{Since both terms in } V(R) \sim \omega^2 \text{, the SUSY-breaking scale is irrelevant for the position of the minimum}}$

The Loop Calculation

needs careful book-keeping, but can be done without new integrals (by thoughtful use of (N=2) SUSY, elementary group theory and Z_2 - trf. properties).

One finds e.g. (for $S^1/(Z_2 \times Z_2')$, with $G \to H$ at one boundary)

$$c^{(1)} \sim \omega^2 (24\, d_G - 56\, d_H)$$

$$+ \text{hypers} + \text{gravity}$$

$$c^{(2)} \sim \omega^2 \frac{2}{\pi^3} C_G (7\, d_G - 15\, d_H)$$

$$+ \text{hypers} \qquad (\text{no gravity since } M \ll M_{P,5})$$

(Note: Calculation can be partially checked against Cheng/Matchev/Schmaltz: "Radiative corrections to KK-masses")

5d SUSY-GUT models

- $S^1/(Z_2 \times Z_2')$; $SU_5 \to SU_3 \times SU_2 \times U_1$ by Z_2'
- Higgs on SM-brane
- Matter from "split multiplets" in bulk or on SM-brane

In detail: $\bar{5}$-matter

$$\bar{5} = \underbrace{(\bar{3},1)}_{u} + \underbrace{(1,2)}_{s}$$

counts # of bulk fields of this type

10-matter

$$10 = \underbrace{(3,2)}_{t} + \underbrace{(\bar{3},1) + (1,1)}_{r}$$

$\Rightarrow \quad r, s, t, u \in (0, 1, 2, 3)$

parameterize our "micro landscape" of $4^4 = 256$ models

- We find: $$\frac{C^{(1)}}{C^{(2)}} = \pi^3 \, \frac{-160 - 16r - 8s + 96t + 48u}{\frac{2424}{5} - 108r - 36s + 36t + 12u}$$

- From this ratio,

  the value of R_{min} and hence α_{GUT} follow.

  (We need $C^{(1)} < 0$, $C^{(2)} > 0$, and small ratio to get large MR.)

$$\boxed{\begin{array}{l}\text{We find a minimum at } MR \gtrsim 10 \text{ for } \sim 1/3 \text{ of the models} \\ \text{We find } \frac{1}{20} \gtrsim \alpha_{GUT} \gtrsim \frac{1}{30} \text{ for 12 models}\end{array}}$$

Uplifting

$N=1$ supergravity perspective:

$$\Omega \sim -(T+\bar{T}) + \Delta\Omega_{loop} \qquad ; \qquad W = W_0$$

$$\Delta\Omega_{loop} \sim \frac{1}{(T+\bar{T})^2} + \frac{g^2}{(T+\bar{T})^3}\ln(M(T+\bar{T}))$$

(cf. "Almost no-scale" proposal of Luty / Okada)

$$V(R_{min}) \sim \frac{\omega^2}{R_{min}^4} \text{ is calculable \& } < 0.$$

To "uplift", allow for a small warping ($\Lambda_5 \neq 0$):

$$\Lambda_5 = -6k^2 M_{P,5}^3 \qquad ; \qquad e^{-k(T+\bar{T})} = 1 - \text{"small"}$$

- Neglecting the loop-induced Kähler-correction for the moment, we have:

$$\Omega \sim e^{-k(T+\bar{T})} - 1 \quad ; \quad W = W_0 \, e^{-3kT}$$

(→ Luty/Sundrum)

(constant superpotential at IR-brane)

- A Kähler-Weyl rescaling gives

$$\Omega \sim 1 - e^{k(T+\bar{T})} \quad ; \quad W = W_0$$

$$\sim -(T+\bar{T}) + \Delta\Omega_W$$

$$\Delta\Omega_W \sim (T+\bar{T})^2$$

- This has to be combined with our $\Delta\Omega_{loop}$

$$\Rightarrow \quad \Delta\Omega_{tot} \sim -M_{P,5}^3 k (T+\bar{T})^2 + \frac{1}{(T+\bar{T})^2} + \frac{g^2}{(T+\bar{T})^3} \ln(M(T+\bar{T}))$$

$$\delta V \sim \frac{|W_0|^2}{M_{P,5}^6} (\Delta\Omega)_{T\bar{T}}$$

$$\delta V \sim \underline{\underline{\text{const.}}} + \frac{1}{R^4} + \frac{g^2}{R^5} \ln(MR)$$

positive
constant
contribution $\sim \omega^2 M_{P,5}^3 k$

- Uplifting to $\Lambda_4 \approx 0$ implies

$$k(T+\bar{T}) \sim \frac{1}{(RM_{P,5})^3}$$

i.e. warping is indeed small and our unwarped calculation is OK.

Note: We have checked explicitly that this is equivalent to "detuning" the IR-brane-tension à la Bagger/Belyaev

(cf. Bagger/Redi '03 ; Falkowski '05 ; Katz/Redi/Shadmi/Shirman '05 Gersdorff/Quiros/Riotto '03 ; Maru/Sakai/Uekusa '06)

In this approach, one simply modifies the (negative) IR-brane tension w.r.t. it RS-model-value.

Note: If one insists on $\Lambda_5 = 0$ (e.g. because one doubts that $\Lambda_5 \neq 0$ can be realized in the heterotic setting), another option is a brane-localized F-term uplift

(cf. Gomez-Reino/Scrucca ; Lebedev/Nilles/Ratz ; Choi/Jeong ; Brümmer/A.H./Trapletti ; Dudas/Mambrini ; ...)

This is consistent with our calculation, but not very elegant...

Summary / Conclusions

- Two-Loop Casimir stabilization occurs naturally in 5d orbifolds (given only F_T-dominance & suitable matter content).

- With a very mild discrete tuning, it can "explain" the smallness of $\alpha_{GUT} \sim 1/25$

- An elegant way of uplifting the stabilized AdS_4-vacua is to appeal to a small warping $(\Lambda_5 \neq 0)$

- It would be interesting to find 2-loop-Casimir-stabil. in explicit heterotic models, in particular to allow for more moduli
(cf. e.g. 6d analysis of Buchmüller / Catena / Schmidt-Hoberg)